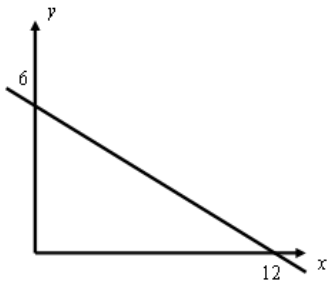
Nat 5 Algebra – Straight Lines, Equations and Algebraic Fractions

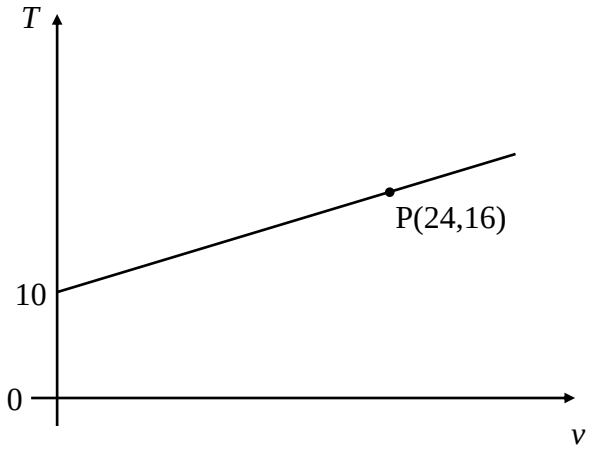
1. A straight line cuts the *x-axis* at the point (12, 0) and the *y-axis* at he point (0,6) as shown.



Find the equation of this line. **3**

2. A line has equation $2y + 6x = 9$.
Find its gradient and *y*-intercept**.** **3**

3. The relationship between variables *v* and *T* produces a straight line graph as shown below.
The line passes through the point P(24,16) as shown.



(a) Find the gradient of the line. **2**

(b) Hence, write down the equation of the line in terms of *v* and *T*. **2**

4. Solve **algebraically** the inequality $2 - 3(x + 5) \leq 7x - 18$ **3**

5. Solve, **algebraically**, the system of equations

$$\begin{aligned} 2x + 3y &= 2 \\ 3x - 4y &= 20 \end{aligned}$$ **3**

6. Jamie and Lachlan both book into the Scotsman Hotel

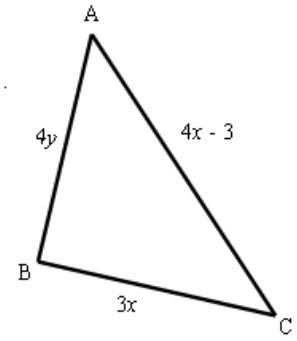
(a) Jamie stays for 3 nights and eats breakfast on 2 mornings.
His bill comes to £312.50
Write down an algebraic equation to illustrate this **1**

(b) Lachlan stays for 5 nights and eats breakfast on 4 mornings
His bill comes to £548.00
Write down an algebraic equation to illustrate this **1**

(c) Find the cost of 2 night's bed **and** 1 breakfast **4**

7. In triangle ABC

AB = $4y$ centimeters
BC = $3x$ centimetres
AC = $4x - 3$



(a) The perimeter of the triangle is 40 cm.

Write down an equation in x and y to illustrate this information. **2**

(b) AB is 7 cm shorter than BC.
Write down another equation in x and y to illustrate this information. **2**

(c) Hence calculate the values of x and y. **3**

8. Change the subject of the formula to p. $E = \frac{V+p}{m}$ **2**

9. Change the subject of the formula $4\sqrt{b} - d = c$ to 'b' **3**

10. A function is defined as $f(x) = 3x - 2$. If $f(t) = 25$, find t **2**

11. Simplify $\frac{2}{x+3} + \frac{3}{x-2}$ **3**

12. Simplify $\frac{5c-5}{c^2-1}$ **3**

Answers

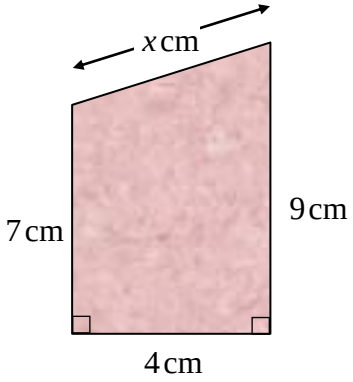
1. gradient is $-\frac{1}{2}$, line is **$y = -\frac{1}{2}x + 6$**
2. rearrange to $y = -3x + 9/2$, gradient is **-3**, y-intercept is **(0, 9/2)**
3. (a) $m = ¼$,
   (b) $y = ¼x + 10$ so **$T = ¼v + 10$**
4. $2 - 3x - 15 \le 7x - 18$, $5 \le 10x$, **$x \ge 1/2$**
5. $8x + 12y = 8$
   $9x - 12y = 60$ $17x = 68$ ***x = 4, y = -2***
6. (a) **3n + 2b = 312.50**
   (b) **5n + 4b = 548**
   (c) one night is £77, one breakfast is £40.75 **Total cost is £194.75**
7. (a) perimeter $4y + 7x - 3 = 40$
   (b) $AB + 7 = 3x$ *so* $4y + 7 = 3x$
   (c) x = 5, y = 2
8. $Em = V + p$, ***p = Em – V***
9. $4\sqrt{b} = c + d$, $\sqrt{b} = \frac{c+d}{4}$, ***b*** $= \left(\frac{c+d}{4}\right)^2$
10. $25 = 3t - 2$, $27 = 3t$ **t = 9**
11. $\frac{2(x-2)+3(x+3)}{(x+3)(x-2)} = \frac{5x+5}{(x+3)(x-2)}$
12. $\frac{5(c-1)}{(c-1)(c+1)} = \frac{5}{c+1}$

Indices & Surds

1. (a) Express in its simplest form: $a^{8} \times (a^{2})^{-3}$ **2**

(b) Remove brackets and simplify: $a^{-\frac{1}{2}}(a^{\frac{1}{2}} - 2)$ **2**

2. The diagram below shows the end view of a scale model of a garden shed.



Calculate the exact value of x, giving your answer as a surd in its simplest form. **4**

3. (a) Simplify $\sqrt{3} \times \sqrt{12}$ **2**

(b) Simplify $\sqrt{3} + \sqrt{12}$ **2**

(c) Hence show that $\dfrac{\sqrt{3} \times \sqrt{12}}{\sqrt{3} + \sqrt{12}} = \dfrac{2\sqrt{3}}{3}$ **2**

Answers

1. (a) expand brackets $a^{8} \times a^{-6}$, simplify $\mathbf{a^{2}}$
(b) expand brackets $a^{0} - 2a^{-1/2}$, simplify $\mathbf{1 - 2a^{-1/2}}$

2 Use Pythagoras $x = \sqrt{(4^2 + 2^2)}$, $x = \sqrt{20}$ $\boldsymbol{x = 2\sqrt{5}}$

3. (a) **6** (b) $\mathbf{3\sqrt{3}}$

(c) rationalize the denominator to get $\dfrac{2}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

Extra help – Straight Lines, Equations and Algebraic Fractions

	Examples to read	Questions to try
1	Equation of a straight line $y = mx + c$ Ex 12.3 Page 96	Q1 Page 97
2	Equation of a straight line $Ax + By + C = 0$ Ex 12.13 - 12.15 Pages 106/7	Q4 Page 108
3	Equation of a straight line Ex 12.4 Page 96	Q4 & 5 Page 99
4	Solving Inequalities Ex 13.9 & 13.10 Pages 116/7	Q2 Page 117
5	Simultaneous Equations Ex 14.8 & 14.9 Pages 126/7	Q2 Page 127
6	Forming and solving simultaneous eq Ex 14.10 Page 128	Q1,2 Page 129
7	Forming and solving simultaneous eq	
8	Changing the subject of the formula ExEx 15.6 – 15.8 Page 134	Q1 Page 135
9	Changing the subject of the formula Ex 15.17 – 15.21 Pages 140/1	Q1,2 Page 142
10	Functions Ex 12.10 & 12.12 Page 104	Q1 – 3 Page 105
11	Adding algebraic fractions Ex 6.3 Page 48	Q2 Page 49
12	Simplifying algebraic fractions Ex 7.5 – 7.7 Page 54	Q2 Page 55

Extra help –Indices and Surds

	Examples to read	Questions to try
1	Using laws of Indices Ex 2.5 – 2.7 Pages 17/18 Ex 2.10 Page 21	Q1 Page 17, Q4 Page 21 Q2 Page 22
2	Exact Values Ex 1.5 Page 7	Q7 & 10 Page 9
3	Simplifying surds Ex 1.3 Page 5, Ex 1.4 Page 6 Ex 1.6 Page 10	Q4 Page 5, Q1 Page 6 Q2 Page 10